NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2208

ANALYSIS OF SHEAR STRENGTH OF HONEYCOMB CORES FOR SANDWICH CONSTRUCTIONS

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SUMMARY

An analysis was undertaken to arrive at a mathematical formula by which the shear strengths of honeycomb cores for sandwich constructions could be calculated. The analysis is partly empirical, being based upon data obtained from previous tests of plywood panels. It was applied successfully to honeycomb cores composed of resin-impregnated papers, but should be verified for materials greatly different before it is generally applied.

A general formula is suggested for use when the cell walls of the honeycomb cores buckle before or at failure. If the cell walls do not buckle, the specific shear strength is approximately constant for cores made of similar materials and having similar cell shapes.

INTRODUCTION

If plates of sandwich construction are designed so that their facings are elastically stable, the most critical stress to which the core is subjected is shear. In a honeycomb type of core construction, a change in the shape or size of the cells or in the type or thickness of the cell walls may be expected to change the strength of the core.

In the present report an analysis was undertaken to arrive at a mathematical formula by which the shear strengths of honeycomb core materials could be calculated. It is assumed that each cell wall acts independently, like a plate supported and loaded along its edges, and that the shear strength of the honeycomb will be determined by the failing stress of these plates. A similar analysis has been completed on the compressive strength of honeycomb cores. (See reference 1.)

Experimental verification of the formula of the present report was obtained by tests of honeycomb-type sheets of resin-impregnated paper (fig. 1); two groups of specimens were tested, each of which represented a different resin-impregnation treatment of the basic paper.

This investigation was conducted at U. S. Forest Products Laboratory under the sponsorship and with the financial assistance of the National Advisory Committee for Aeronautics.

DERIVATION OF FORMULA

It is assumed that each cell wall of the honeycomb structure will act independently, like a plate supported and loaded along its edges, and that the shear strength of the honeycomb will be determined by the failing stress of the plates.

The critical buckling stress of such a plate is expressed by the following formula (reference 2):

$$\tau_{cr} = kE\frac{h^2}{a^2} \tag{1}$$

where

τ_{cr}	critical buckling stress of plate in shear, psi
a	width of plate, inches
h	thickness of plate, inches
E	modulus of elasticity of plate material, psi
k	constant depending on type of edge support and directional properties of plate

For the formula to be generally applicable to honeycomb constructions, the value of k must take into account the narrow walls of double thickness at the junctions of the corrugations, the wider walls of single thickness, and the fact that the wider walls may be curved rather than flat.

As for plates in compression, the failing stress generally exceeds the critical buckling stress. (See reference 1.) It can be shown that for plywood plates a good approximation of the data is given by the equation

$$\frac{\tau_f}{\tau_u} = \left(\frac{\tau_{cr}}{\tau_u}\right)^{1/2} \tag{2}$$

where

τ_f average shear stress of plate at failure due to buckling, psi

τ_u shear strength of plate material, psi

τ_{cr} critical buckling stress of plate in shear, psi

Equation (2) is valid only if the computed critical stress τ_{cr} is less than the proportional limit of the material.

Figure 2 is a plot of the results and the empirical curve of plywood plates tested in shear in reference 3. The data are plotted on rectangular coordinates. The ordinate is the ratio of the average shear stress at failure due to buckling to the shear strength of the material τ_f/τ_u, and the abscissa a nondimensional parameter of panel width a/a_o. The value a_o is the width of a panel which will fail just as the panel buckles. Thus from equation (2):

$$\frac{a}{a_o} = \left(\frac{\tau_u}{\tau_{cr}}\right)^{1/2} \tag{3}$$

By using this equality, a second empirical curve was superimposed on the experimental data by employing equation (2). It can be seen from figure 2 that the plotted points fall reasonably well around this curve except at high values of τ_f/τ_u, where the critical stress often exceeds the stress at the proportional limit of the material.

Equation (2) can be expanded to

$$\begin{aligned} \tau_f &= \left(\tau_{cr}\tau_u\right)^{1/2} \\ &= \frac{h}{a}\,(kE)^{1/2}(\tau_u)^{1/2} \end{aligned} \tag{4}$$

and the specific shear strength of the material is

$$\tau_s = \frac{h}{ga}\,(kE)^{1/2}(\tau_u)^{1/2}$$

where g is the specific gravity of the material. Also, τ_s may be considered the specific shear strength of a honeycomb construction, since

$$\frac{\tau_a}{g_a} = \frac{\tau_f}{g} = \tau_s$$

where τ_a is the apparent shear strength in pounds per square inch and g_a is the apparent specific gravity of the core structure.

Following the reasoning outlined in reference 1, it can be seen that the shear stress is dependent upon the thickness and width of the cell wall. Since the plates in many honeycomb cores are not flat, it is impossible to determine the proper widths of the individual plates. This width, however, can be considered proportional to any cross-sectional dimension of the cell. The proportionality factor will be different for cells of different shape, but will not change with cell size or cell-wall thickness. For purposes of convenience in connection with cores made of corrugated sheets cemented together, the plate width a will be considered proportional to the height of the corrugation α; that is,

$$a = n\alpha \tag{5}$$

It is difficult to measure accurately the thickness of the cell walls in a honeycomb core. This thickness, however, can be expressed in terms of the apparent specific gravity of the core and the specific gravity of the material from which the core is made. The apparent specific gravity can be calculated from the weight and gross dimensions of a piece of the core, and the specific gravity of the material can be calculated from the weights of a piece of the core in air and submersed in a liquid.

Figure 1 is a sketch of a section of a half cell, or one complete corrugation of the corrugated material used in the manufacture of the core. The weight of this section is

$$w = ru\alpha hbgq$$

where q is the density of water, and r, u, α, h, and b are as shown in the figure.

The gross volume of the piece shown in figure 1 is

$$v = (\alpha + h)u\alpha b$$

and the apparent specific gravity

$$g_a = \frac{w}{vq} = \frac{rhg}{\alpha + h} \tag{6}$$

Then

$$\frac{h}{\alpha} = \frac{g_a}{rg - g_a} \tag{7}$$

in which r is the ratio of the developed (original) length of the corrugated sheet to the length of the sheet after corrugation. This ratio can be determined in a number of ways.

Combining equations (4) and (5),

$$\tau_f = \frac{h}{n\alpha} (kE)^{1/2}(\tau_u)^{1/2}$$

The constants n and k are related to the shape of the cells, so that they may be combined into a single constant c'

$$\tau_f = \frac{h}{\alpha} c'E^{1/2}(\tau_u)^{1/2}$$

and the specific shear strength for specific gravity g is

$$\tau_s = \frac{h}{\alpha} \frac{c'E^{1/2}(\tau_u)^{1/2}}{g} \tag{8}$$

If all the honeycomb cores are made of the same material, g, E, and τ_u remain constant and can be combined with c' to form a single constant C.

Equation (8) can then be combined with equation (7) into the simple form

$$\tau_s = C \frac{g_a}{rg - g_a} \tag{9}$$

where

$$C = \frac{c'(E)^{1/2}(\tau_u)^{1/2}}{g}$$

and can be evaluated from experiments in which the other quantities in equation (9) have been measured.

The value of C will remain constant even for different materials, provided the modulus of elasticity and the proportional limit vary directly with the specific gravity. This is approximately true for a paper impregnated with a resin. If the resin content varies over a limited range, the modulus of elasticity and the proportional limit will be roughly proportional to the specific gravity.

The value of C is useful in the comparison of two honeycomb cores of different cell shapes and made of different materials of sizes such that the cell walls buckle before failure. If specimens of two such cores do not have identical apparent specific gravities, a comparison of their specific shear strengths is not proper because the apparent shear strength does not vary directly with the specific gravity. A comparison of the values of C for the two cores is, however, accurate, inasmuch as such a comparison yields a ratio identical to that which would be obtained if specimens of like specific gravities were compared. The dimensions of C are those of a specific stress, and therefore C might be called the fundamental specific shear stress.

If the cores are such that the cell walls do not buckle before failure, the constant C should not be used as a comparison of the two materials. In such a case, the apparent shear strength of each core will vary directly with the apparent specific gravity of the material. If the two cores have identical apparent specific gravities, the ratio of their apparent shear strengths will be the same as the ratio of their specific shear strengths.

EXPERIMENTAL VERIFICATION

Eleven blocks of honeycomb core material were fabricated for the tests to verify the formula. Several thicknesses of paper and two cell sizes were employed to obtain a suitable range in apparent specific gravity among the blocks. The first group of six blocks was a comparatively low-strength series, selected so that buckling of the cell walls would occur before failure in some of the constructions. The second group of five blocks was of higher strength, in which no buckling occurred.

The core material for group 1 was made from four thicknesses of kraft paper. The paper was treated with about 20 percent by weight (based on the treated sheet) of water-soluble phenol resin and corrugated with A- or B-flute corrugations. The sheets were then bonded crest to crest with an acid-catalyzed phenol-resin adhesive with a spread of about 2 to 3 grams of adhesive per square foot of corrugated paper. The core was then placed in an oven at a temperature of 125° C to cure the resin.

The core material for group 2 was made from three thicknesses of kraft paper. The paper was first pretreated with about 10 percent by weight of water-soluble phenol resin and corrugated with either A- or B-flute corrugations. The sheets were then treated with 50 to 55 percent by weight (based on the cured treated core) of a high-temperature-setting, low-viscosity, laminating resin of the polyester type, and were bonded crest to crest into blocks of honeycomb material. The core was subjected to a temperature of 135° C to cure the resin.

Specimens were cut perpendicular to the direction of the cells from the finished honeycomb blocks, to a thickness of 0.500 ± 0.005 inch. The specimens were trimmed to 2.00 by 6.00 inches and bonded to shear plates. Shear was applied by the tension-frame method described in reference 4. Figure 3 is a detailed drawing of the tension specimen. The specimens were made so that shear deformation resulted in the LT-plane for some specimens and in the LR-plane for others (fig. 4). For specimens to be tested in the LR-plane, three widths of core material were bonded crest to crest in order to obtain a specimen of sufficient length.

Failure of the A-flute type of specimens of group 1, in both the LT- and LR-planes, was caused by buckling of the cell walls, followed by either complete collapse of the cells or a shear failure of the core parallel to the shear plate. There was no evidence of buckling in either type of B-flute specimens. Failure of specimens tested in the LT-plane was primarily due to shear failure parallel to the plate, while failure of specimens tested in the LR-plane was due to a combination of shear failure of the material and of the bonds between the corrugated sheets. The specimens of group 2 showed no evidence of buckling before or at the maximum stress, and the primary failure of these specimens was due to diagonal tension of the core.

Test data and pertinent information about each block of core material are given in table 1.

When the specimen fails by buckling of the cell walls, as in group 1, equation (9) may be applied as follows:

$$C = \tau_s \frac{rg - g_a}{g_a} \tag{10}$$

Substituting appropriate values from table 1 in the above equation, the constants for blocks tested in the LR-direction (subscripts denote block numbers) are

$$\begin{aligned}
C_{31} = 710 \frac{1.35 \times 0.789 - 0.0236}{0.0236} &= 31,300 \\
C_{32} &= 33,000 \\
C_{33} &= 30,200 \\
C_{34} &= 25,200 \\
C_5 &= \underline{23,900} \\
\text{Average} \quad & 28,700
\end{aligned}$$

The value of C_5 is included in the average because, although no buckling was noted, the value is similar to the others, a fact that indicates that this particular construction is close to the point where buckling may or may not take place.

Similarly, for specimens tested in the LT-direction:

$$\begin{aligned} C_{31} &= 39{,}700 \\ C_{32} &= 53{,}700 \\ C_{33} &= 50{,}800 \\ C_{34} &= 42{,}400 \\ C_5 &= \underline{41{,}200} \\ \text{Average} \quad & 45{,}600 \end{aligned}$$

Again C_5 is included for the reason mentioned above.

The values of C are reasonably constant within each group, a fact that indicates that equation (9) is a reasonable one. The higher value of C obtained from tests in the LT-direction indicates that the shear strength in that plane is approximately 60 percent higher than the shear strength of a comparable specimen in the LR-plane. Examination of this type of honeycomb structure shows that such a difference is to be expected.

Thus the equation

$$\tau_s = 28{,}700 \frac{g_a}{rg - g_a} \tag{11}$$

can be applied to this type of honeycomb material subjected to shear strains in the LR-plane. The equation

$$\tau_s = 45{,}600 \frac{g_a}{rg - g_a} \tag{12}$$

can be applied to this type of honeycomb material subjected to shear strains in the LT-plane.

For design purposes, equation (11) or (12) can be applied to honeycomb cores that buckle in order to determine what apparent specific gravity is required to obtain a desired specific shear strength. Figure 5 shows the relationship between the apparent shear strength and the apparent specific gravity of the honeycomb structures of group 1. It is evident that the equations are applicable only in the buckling range, and at higher strength values the apparent shear strength varies directly with the apparent specific gravity.

In case the cell walls do not buckle before failure, as in group 2, the apparent shear strength is directly proportional to the apparent specific gravity; that is, the specific shear strength is approximately constant. This is illustrated by the specific-shear-strength values of specimens from group 2 (table 1) or by the plotted data (fig. 6).

The experimental values obtained from the tests are in reasonable agreement with the curves in figures 5 and 6. The deviation of the experimental values is probably due largely to the variation in the specific gravity of the impregnated papers making up the core material and to the variation in the values of r for the individual specimens.

From the data discussed above, a paper-honeycomb core material can be designed to meet specific requirements. If a low-strength material is desired, that is, one that will buckle before failure, equation (9) can be applied to arrive at a calculated value. Equations (11) and (12) will probably be valid for honeycomb cores made of this type of material and having cell shapes similar to those of the cores tested. If a different material is used, a few tests can be made to determine the value of the applicable constant, and this equation can be used for design purposes. If the honeycomb core material is such that buckling does not occur before failure, the apparent shear strength will vary directly as the apparent specific gravity.

Forest Products Laboratory
Madison, Wis., June 17, 1949

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2. March, H. W.: Buckling of Flat Plywood Plates in Compression, Shear, or Combined Compression and Shear. Mimeo. No. 1316, Forrest Products Lab., U. S. Dept. Agric., April 1942.

3. Norris, C. B., Voss, A. W., and Palma, Joseph, Jr.: Buckling of Flat Plywood Plates in Uniform Shear with Face Grain at 0°, 45° and 90°. Mimeo. No. 1316-H, Forest Products Lab., U. S. Dept. Agric., Feb. 1945. Supp. to "Buckling of Flat Plywood Plates in Compression, Shear, or Combined Compression and Shear."

4. Anon.: Methods of Test for Determining Strength Properties of Core Material for Sandwich Construction at Normal Temperatures. Rep. No. 1555, Forest Products Lab., U. S. Dept. Agric., revised Oct. 1948.

TABLE 1.- PROPERTIES AND DIMENSIONS OF KRAFT-PAPER HONEYCOMB CORE MATERIAL TESTED

Core	Number of tests	Direction of test (a)	Corrugation flute type	Apparent shear strength, τ_a (psi)	Specific shear strength, τ_s (psi)	Apparent specific gravity of material, g_a	Specific gravity of material, g	Thickness of material (approx.), h (in.)	Height of corrugations plus thickness of material, α + h (in.)	Ratio of corrugation height to wave length, 1/n	Ratio of original length of corrugated sheet to length of sheet after corrugation, r
Group 1 - Cores impregnated with 20 percent water-soluble phenolic resin											
31	5	LR	A	16.7	710	0.0236	0.828	0.002	0.161	0.39	1.29
	5	LT	A	21.3	900	.0236	.828	.002	.161	.39	1.29
32	3	LR	A	27.6	940	.0295	.779	.003	.169	.42	1.36
	5	LT	A	45.1	1530	.0295	.779	.003	.169	.42	1.36
33	3	LR	A	42.1	1110	.0378	.775	.005	.176	.45	1.39
	4	LT	A	70.8	1870	.0378	.775	.005	.176	.45	1.39
34	3	LR	A	64.0	1260	.0507	.745	.008	.180	.47	1.44
	5	LT	A	107.4	2120	.0507	.745	.008	.180	.47	1.44
5	6	LR	B	95.6	1510	.0632	.846	.005	.090	.32	1.32
	4	LT	B	164.5	2600	.0632	.846	.005	.090	.32	1.32
36	3	LR	B	133.6	1570	.0850	.761	.008	.094	.34	1.28
	5	LT	B	235.9	2780	.0850	.761	.008	.094	.34	1.28
							[b].789			[b].40	[b]1.35
Group 2 - Cores impregnated with 50 to 55 percent low-viscosity polyester resin											
12	5	LR	A	178	2370	0.075	1.27	0.005	0.142	0.37	1.28
	5	LT	A	256	3410	.075	1.27	.005	.142	.37	1.28
13	5	LR	A	336	2780	.121	1.29	.009	.136	.35	1.21
	5	LT	A	428	3540	.121	1.29	.009	.136	.35	1.21
14	5	LR	A	449	2610	.172	1.27	.012	.127	.32	1.19
	5	LT	A	670	3900	.172	1.27	.012	.127	.32	1.19
15	5	LR	B	464	2750	.169	1.30	.009	.098	.36	1.27
	6	LT	B	611	3620	.169	1.30	.009	.098	.36	1.27
16	5	LR	B	631	2650	.238	1.28	.013	.095	.32	1.19
	6	LT	B	827	3480	.238	1.28	.013	.095	.32	1.19
							[b]1.28			[b].34	[b]1.23

[a]Specimens tested to produce shear deformation in either LR- or TR-plane. (See fig. 4.)
[b]Average of values in group.

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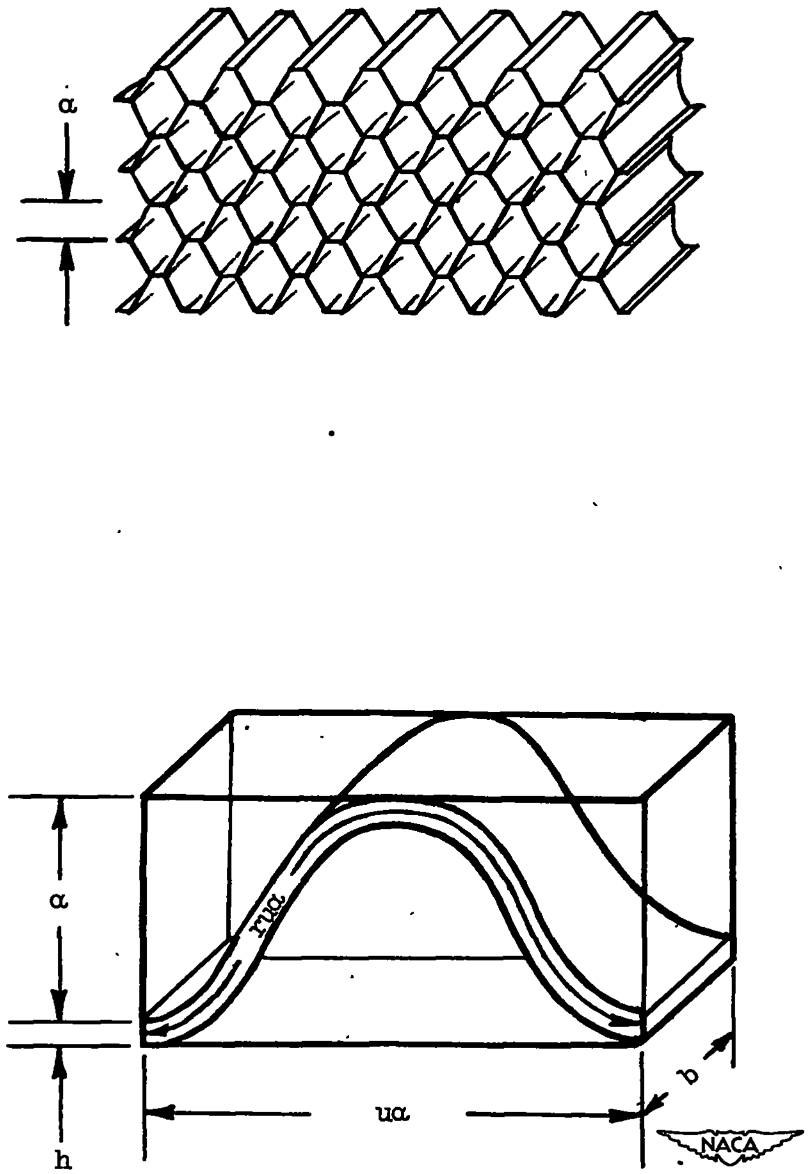

Figure 1.- Sketch of honeycomb core material and of one corrugation.

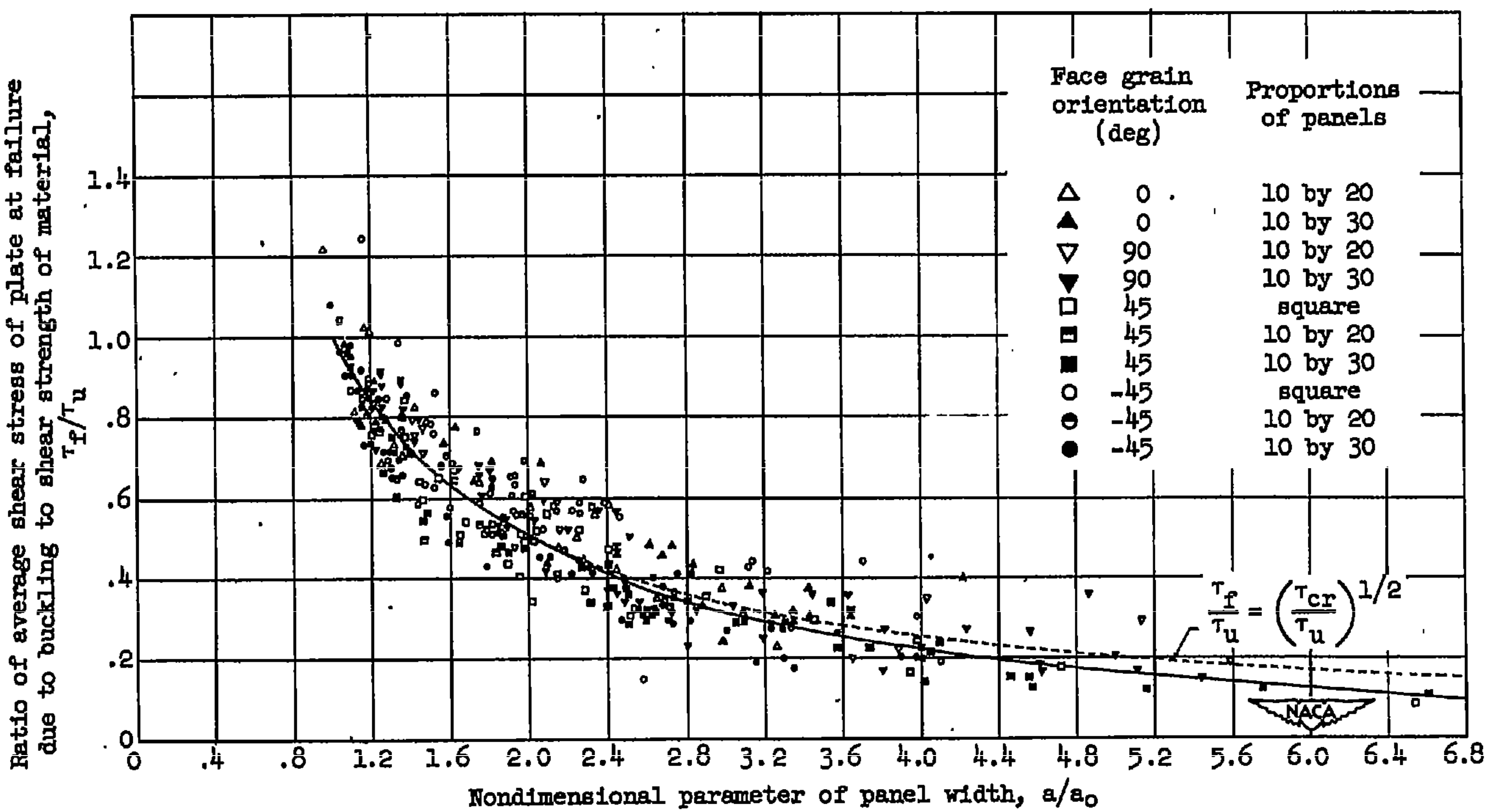

Figure 2.- Composite effective ultimate stress for plywood panels tested in shear. Data taken from reference 3.

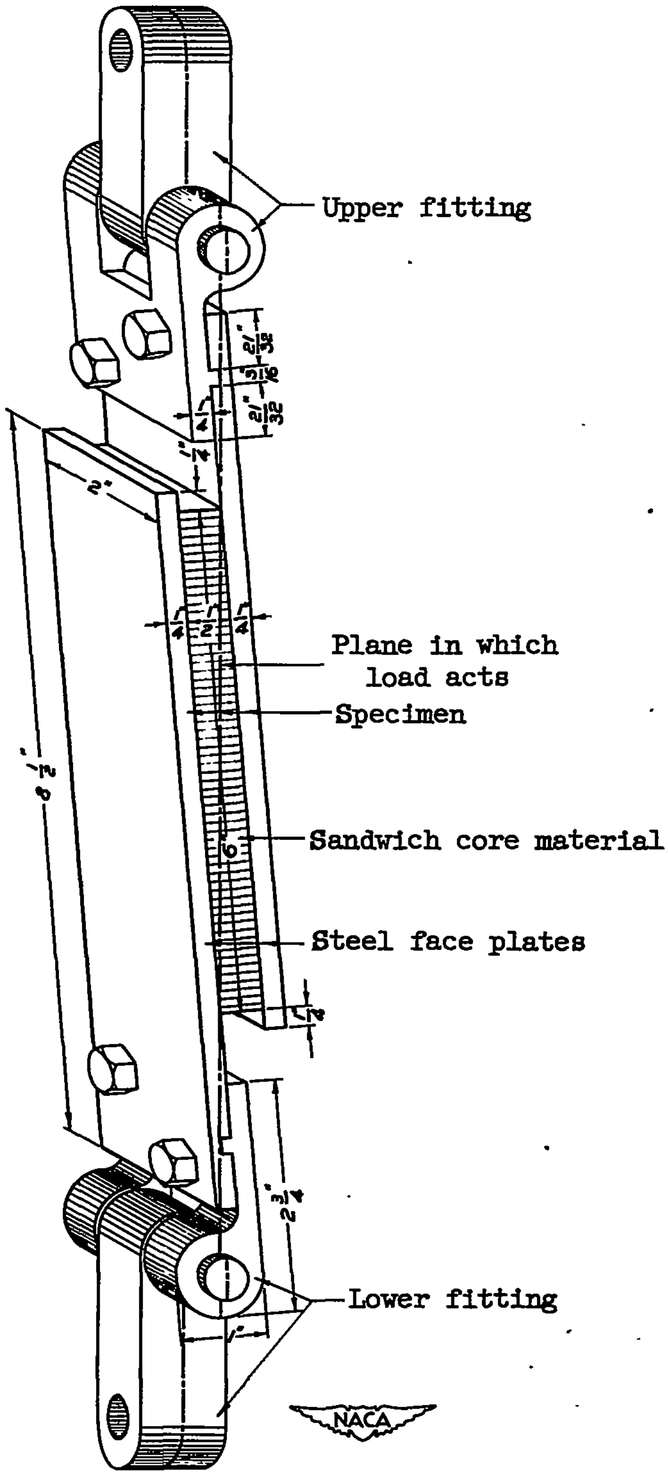

Figure 3.- Detail of tension-frame shear specimen used for shear tests of sandwich core material.

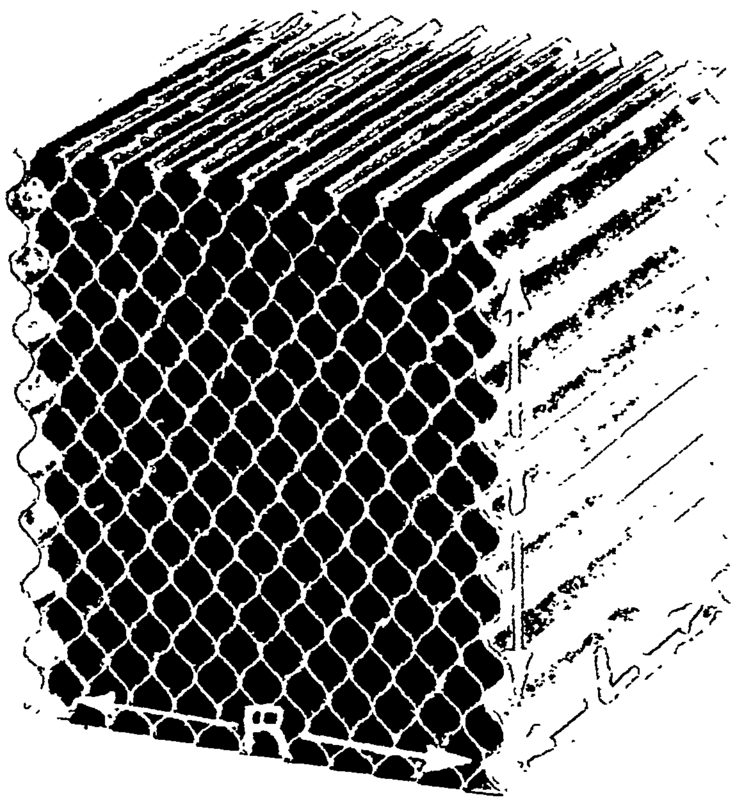

Figure 4.- Paper-honeycomb block showing directional orientations, referred to as L (longitudinal), R (radial), and T (tangential).

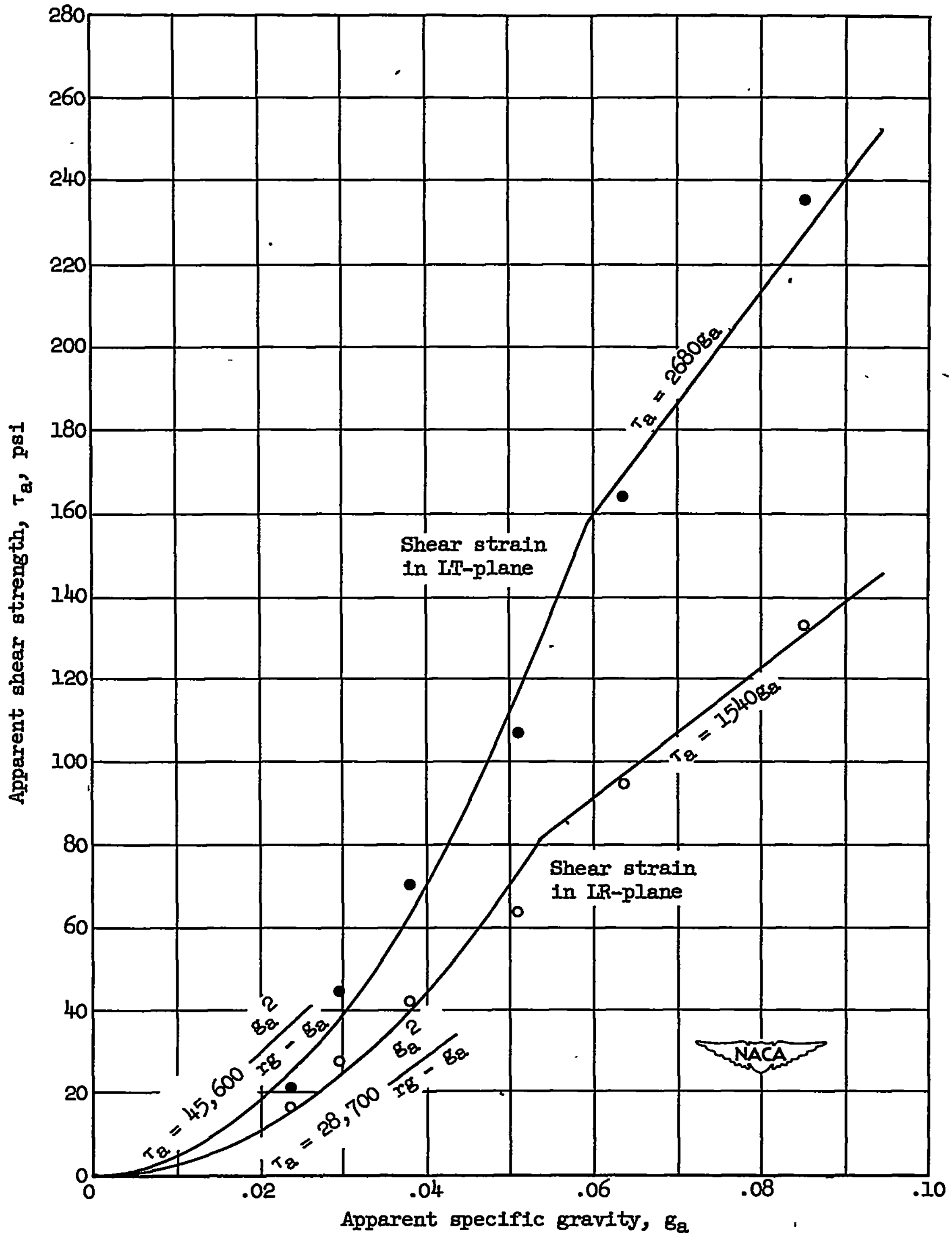

Figure 5.- Apparent shear strength of paper honeycomb cores against apparent specific gravity. Initial part of curve where cell walls buckle is based on formula $\tau_s = C \frac{g_a}{rg - g_a}$.

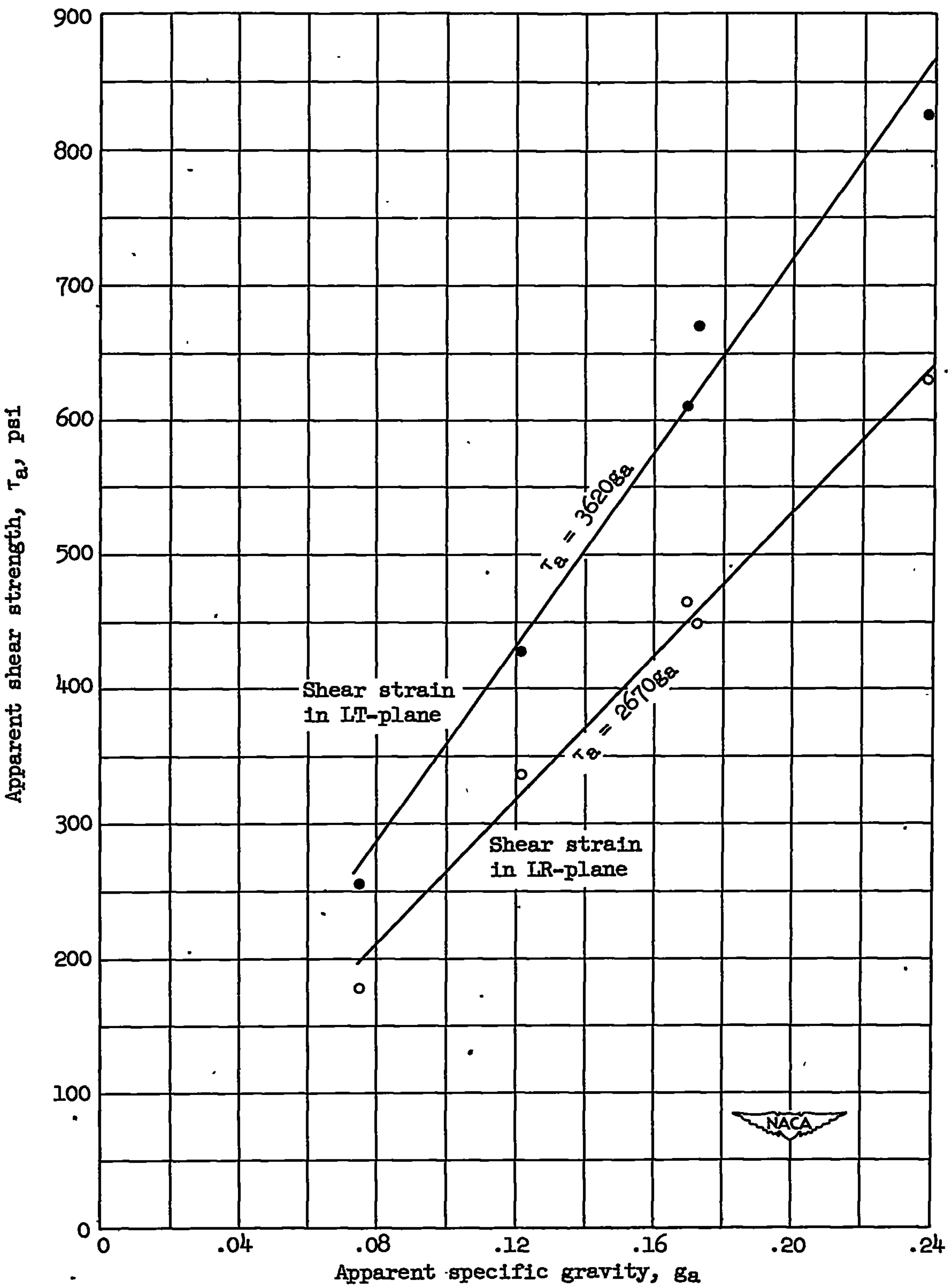

Figure 6.- Apparent shear strength of paper honeycomb cores against apparent specific gravity. Cell walls of cores did not buckle.